P-ADIK RATSIONAL DINAMIK SISTEMALAR

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Annotatsiyasi:Ishni bajarishda matematik analiz, kompleks analiz va p-adik analiz usullaridan foydalanildi. P-Adik kompleks sonlar maydoni ustida berilgan diskret vaqtli dinamik sistemalar nazariyasini o'rganish va ularni qo'llab:

- *a/(x^2+1) –ratsional funksiya qo'zg'almas nuqtalarini topish;*
- *Qo'zg'almas nuqta yagona bo'lgan holda bu qo'zg'almas nuqtaning xarakteriga mos ravishda dinamikani tadqiq etish;*
- *Qo'zg'almas nuqta mavjud bo'lmagan holda davriy nuqtalarni topish va bu davriy nuqtalarning xarakteriga mos ravishda dinamikani tadqiq etish.*

Maqola natijalari p-adik ratsional funksiyalar bir sinfi uchun diskret dinamik sistemalari nazariyasini rivojlantirishda qo'llaniladi.Natijalarining amaliy ahamiyati telekomunikatsiyaning ba'zi masalalarini va raqamli tahlil hamda kriptografiyada qo'llash imkoniyati bilan izohlanadi. Asosiy natijalar nazariy xarakterga ega. Matematikaning turdosh sohalari va biologik va fizik sistemalar dinamikasini o'rganishda qo'llanadi.

Kalit so`zlar: Ratsional dinamik sistemalar; qo`zg'almas nuqta; invariant to'plam; Siegel disk; p-adik kompleks sonlar maydoni.

P-АДИК РАЦИОНАЛЬНЫЕ ДИНАМИЧЕСКИЕ СИСТЕМЫ

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Аннотация: В работе использованы методы математического анализа, комплексного анализа и p-адического анализа. Изучение теории динамических систем с дискретным временем в области p-адических комплексных чисел и их реализация:

- *определить неподвижные точки a/(x^2+1) -рациональной функции;*
- *Реализация динамики, соответствующей характеристикам фиксированной точки, когда фиксированная точка единственно;*
- *найти периодические точки при отсутствии фиксированной точки и реализовывать динамику в соответствии с природой этих периодических точек.*

Результаты работы по развитию теории дискретных динамических систем для класса p-адических рациональных функций, имеют практические применение в рассмотрения проблем телекоммуникаций и возможностью их использования в цифровом анализе и криптографии. Основные результаты имеют теоретический характер. Они используются в смежных областей математики и динамики биологических и физических систем.

Ключевые слова: Рациональные динамические системы; фиксированная точка; инвариантное множество; Диск Зигеля; Множество p-адических комплексных чисел.

P-ADIC RATIONAL DYNAMIC SYSTEMS

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Annotation: The work uses the methods of mathematical analysis, complex analysis and p-adic analysis. Study of the theory of dynamical systems with discrete time in the field of p-adic complex numbers and their implementation:

- Find fixed points of a/(x^2+1) - rational functions;
- study the dynamics corresponding to the characteristics of fixed and points, in particular when this point is unique;
- find periodic points in case of the absence of a fixed point and investigate the dynamics in accordance with the nature of these points;

The results of the work on the development of the theory of discrete dynamical systems for the class of p-adic rational functions. The practical significance of the results obtained is expressed in the fact that they can be used for the further development of telecommunications, digital analysis and cryptography. The results have theoretical character. They can be used in the study of related areas of mathematics, the dynamics of biological and physical systems.

Key words: Rational dynamical systems; fixed point; invariant set; Siegel disk; fields of p-adic complex numbers.

Ma'lumki, p-adik sonlar nazariyasi ko'plab qo'llanmalarga ega. Masalan, matematika, biologiya, fizika va boshqa fanlarning tarmoqlarida p-adik sonlardan keng foydalanilgan.

Quyidagi asosiy ta'riflardan eslatamiz. Eng katta umumiy bo'luvchini (n, m) bilan belgilaymiz, bunda n va m musbat butun sonlarning. Q ratsional sonlar maydoni bo'lsin.

Har bir tub son p uchun ixtiyoriy ratsional $x \neq 0$ sonni quyidagicha ifodalash mumkin (p, n) = 1, (p, m) = 1. Bu x ning p-adik normasi | $|x|_p = p^{-r}$ va $|o|_p = 0$ bu yerda r, n∈Z, m musbat butun son.

Ushbu norma quyidagi xossalarga ega:

1) $|x|_p \geq o$ va $|x|_p = o$ agar faqat x = 0 bo'lsa,

2) $|xy|_p = |x|_p |y|_p$

3) kuchli uchburchak tengsizligi

$$|x+y|_p \leq \max\{|x|_p, |y|_p\}.$$

3.1). agar $|x|_p \neq |y|_p$ bo`lsa $|x+y|_p = \max\{|x|_p, |y|_p\}$,

3.2) . agar $|x|_p = |y|_p$ bo`lsa p=2 uchun $|x+y|_p \leq \frac{1}{2}|x|_p$

Q ning p-adik normaga nisbatan to'ldirilgani p-adik sonlar maydonini aniqlaydi va Q_p bilan belgilanadi. Q_p ning algebraik to'ldirmasi C_p bilan belgilanadi va u p-adik kompleks sonlar to'plami deyiladi.

Har qanday a∈ C_p va r> 0 uchun ochiq shar, yopiq shar va sfera quyidagicha belgilanadi

$$U_r(a) = \{x \in C_p : |x-a|_p < r\}$$

$$V_r(a) = \{x \in C_p : |x-a|_p \leq r\}$$

$$S_r(a) = \{x \in C_p : |x-a|_p = r\}.$$

Dinamik tizimni aniqlash uchun biz $f : x \in U \rightarrow f(x) \in U$ funksiyasini qaraymiz, (biz qarayotgan holatda U = U_r (a) yoki C_p).x ∈ U uchun $f^n(x)$ bilan f ning o'zi-o'ziga n-katlama iteratsiyasi belgilanadi:

$$f^n(x) = f(f(f(f \ldots f(x))))\ldots)$$

Ixtiyoririy tanlangan x_0 ∈ U va f: U → U uchun diskret vaqtli dinamik sistemasi (shuningdek, traektoriya deb ataladi) quyidagi ketma-ketlik bilan aniqlanadi

$$x_0, x_1 = f(x_0), \quad x_2 = f^2(x_0), \ x_3 = f^3(x_0), \ldots \quad (3.1.1)$$

Asosiy muammo: f funksiyasi va x_0 boshlang'ich nuqtasi berilganda (3.1.1) ketma-ketlikda nima sodir bo'ladi. $\lim\limits_{n\to\infty} x_n$ limiti mavjudmi? Agar yo'q bo'lsa ketma-ketlikning limit nuqtalari to'plami qanday?

Agar f(x) = x bo'lsa, x ∈ U nuqta f uchun qo'zg'almas nuqta deyiladi. Barcha qo'zg'almas nuqtalar to'plami Fix (f) bilan belgilanadi.

x nuqta m davriy nuqtasidir, agar $f^m(x) = x$.

Agar U (x_0) ning atrofi mavjud bo'lsa, x_0 qo'zg'almas nuqtasi tortuvchi nuqta deb ataladi agar barcha $x \in U(x_0)$ nuqtalar uchun quyidagi tenglik o'rinli bo'lsa

$$\lim_{n\to\infty} f^n(x) = x_0.$$

Agar x_0 tortuvchi bo'lsa, unda uning jalb havzasi quyidagicha aniqlanadi

$$A(x_0) = \{x \in C_p : f^n(x) \rightarrow x_0, n \rightarrow \infty\}$$

Agar x_0 ning U(x_0) atrofi mavjud bo'lib

$x \in U(x_0)$, $x \neq x_0$ uchun $|f(x) - x_0|_p > |x - x_0|_p$

bajarilsa sobit x_0 nuqta repeller deb ataladi.

U_r (x_0) to'pi Siegel disk deyiladi, agar har bir shar $S_\rho(x_0)$, $\rho > r$ funksiya f(x) ga nisbatan invariant bo'lsa, ya'ni agar $x \in S_\rho(x_0)$ bo'lsa, u holda

$f^n(x) \in S_\rho(x_0)$, $n = 1,2\ldots$

Siegel disk SI (x_0) bilan belgilanadi.

Chiziqli bo`lmagan $f(x) = \frac{a}{x^2}$ funksiyaning p-adik dinamik sistemalari.

Ma`lumki p-Adik norma $|.|_p$ ga nisbatan Q ratsional sonlar to`plamining to'ldirmasi (barcha limit nuqtalari to'plami) Q_p orqali belgilanadigan p-adik maydonni aniqlaydi.

Q_p ning algebraik to'ldirmasi kompleks p-adik sonlar maydoni deb ataladi va u C_p kabi belgilanadi.

Ixtiyoriy $a \in C_p$ va r>0 uchun quyidagi to'plamlarni qaraylik

$U_r(a) = \{x \in C_p : |x - a|_p < r\}$,

$V_r(a) = \{x \in C_p: |x - a|_p \leq r\}$,

$S_r(a) = \{x \in C_p: |x - a|_p = r\}$.

Faraz qilaylik x_0 nuqta biror $f(x)$ funksiyaning qo`zg`almas nuqtasi bo`lsin, ya'ni $f(x_0)=x_0$.

$\lambda = f'(x_0)$ bo`lsin. Agar $0 < |\lambda|_p < 1$, bo`lsa x_0 ga tortuvchi nuqta, $|\lambda|_p =1$ bo`lsa netral nuqta va $|\lambda|_p >$ 1 bo`lsa itaruvchi nuqta deyiladi .

U_r (x_0) shar Siegel diski deyiladi, agar uning har bir sferasi $S_p(x_0)$, $\rho < r$ f(x) ning invariant sferasi bo`lsa, yani agar $x \in S_p(x_0)$ bo`lsa, u holda $f^n(x) \in S_p(x_0)$ bo`ladi bu yerda n = 1, 2 . . . . Barcha Siegel disklarining ($x_0$ markazli) birlashmasi maksimal Siegel diski deyiladi va SI ($x_0$) kabi belgilanadi . $f$ funksiyaga bog`liq dinamik sistemani qaraymiz: $f : C_p \to C_p$ bu Sistema quyidagicha aniqlangan:

$f(x) = \frac{a}{x^2}$, a ≠ 0, $a \in C_p$, (1)

bu yerda $x \neq 0$

Bizning maqsadimiz C_p – kompleks p-adik maydonda , (1) ning $\{f^{(n)}(x),\ x \in C_p\}$ trayektoriyalarining xossasini o`rganish .

Eslatma: $f(f(x)) = x$.

Haqiqatdan : $f(f(x)) = \frac{a}{(\frac{a}{x^2})^2} = x$

Ushbu funksiya uchta qo'zg'almas nuqta x_k, k=1, 2, 3 ga ega, ular C_p da $x^3 = a$ yechimlari. Ushbu nuqtalari uchun

$x_k^3 = a \Rightarrow |x_k^3|_p = |a|_p \Rightarrow |x_k|_p = \alpha = (|a|_p)^{1/3}$. (2)

Shunday qilib, $x_k \in S\alpha(0)$, k = 1, 2, 3.

3.1.1 Lemma. (2) tenglik bilan aniqlangan α uchun quyidagi tasdiqlar orinli bo'ladi:

1. $S\alpha(0)$ sfera f ga nisbatan invariant, (ya'ni $f(S\alpha(0)) \subset S\alpha(0)$);
2. $f(U\alpha(0)) \subset C_p \setminus V\alpha(0)$;
3. $f(C\alpha \setminus V\alpha(0)) \subset U\alpha(0)$.

Quyidagicha tenglikni hosil qilamiz:

$f'(x) = \frac{-2a}{x^3} = \frac{-2}{x} \cdot f(x)$.

Qo'zg'almas nuqtalarda

$f(x_k) = \frac{-2}{x_k} \cdot f(x_k) = -2$

$$|f'(x_k)|_p = \begin{cases} \frac{1}{2}, \text{agar } p = 2 \\ 1, \text{ agar } p \geq 3 \end{cases}$$

munosabatlar o'rinli bo'ladi.

Demak, x_k qo'zg'almas nuqta p=2 uchun tortuvchi, $p \geq 3$ uchun esa neytral nuqta bo'ladi.

Bu yerda biz f^n ni aniq hisoblashimiz mumkin:

3.1.2 Lemma. Barcha x {Cp \ {0} uchun

$f^n(x) = a^{1/3(1-(-2)^n)} \cdot x^{(-2)^n}$, n≥1

munosabat o'rinli bo'ladi.

Ta'kidlash joizki $\alpha = (|a|_p)^{1/3}$. r>0 uchun, $x \in S_r(0)$ nuqtani qaraymiz, ya'ni, $|x|_p = r$. Bundan

$|f^n(x)|_p = \left|a^{1/3(1-(-2)^n)} \cdot x^{(-2)^n}\right|_p = a^{1-(-2)^n} \cdot r^{(-2)^n}$, n≥1. (3)

Tanlangan r>0 uchun $r_n = a^{1-(-2)^n} \cdot r^{(-2)^n}$ belgilashni kiritamiz.

So'ngra (3) munosabatdan $x \in S_r(0)$ ning $f^n(x)$, n≥1 traektoriyasi quyidagi sferalar ketma-ketligiga ega:

$S_r(0) \to S_{r1}(0) \to S_{r2}(0) \to S_{r3}(0) \to \ldots$

Endi biz r_n ning limitini hisoblaymiz.

n juft bo'lsin. (3) dan quyidagi munosabatni hosil qilamiz:

$$\lim_{n\to\infty}|f^n(x)|p = \lim_{n\to\infty} r_n = \begin{cases} 0, & \text{agar } r < \alpha \\ \alpha, & \text{agar } r = \alpha \\ +\infty, & \text{agar } r > \alpha \end{cases}$$

n toq bo'lsin. Bu holda quyidagi munosabatga ega bo'lamiz:

$$\lim_{n\to\infty}|f^n(x)|p = \lim_{n\to\infty} r_n = \begin{cases} +\infty, & \text{agar } r < \alpha \\ \alpha, & \text{agar } r = \alpha \\ 0, & \text{agar } r > \alpha \end{cases}$$

Yuqorida keltirilgan natijalarni umumlashtirib, quyidagi teoremani bayon qilamiz:

3.1.1 Teorema. Agar p≥3 va α (2) tenglik bilan aniqlangan bo'lsa, u holda quyidagi munosabatlar o'rinli bo'ladi.

1.Agar $x \in U\alpha(0)$ bo'lsa, u holda

$\lim_{k\to\infty} f^{2k}(x) = 0$, $\lim_{k\to\infty}|f^{2k-1}(x)|p = +\infty$

bo'ladi;

2. Agar $x \in S\alpha(0)$ bo'lsa, u holda $f^n(x) \in S\alpha(0)$, $n \geq 1$ bo'ladi;

3. Agar $x \in Cp \setminus V\alpha(0)$ bo'lsa, u holda

$\lim_{k\to\infty} f^{2k-1}(x) = 0$, $\lim_{k\to\infty}|f^{2k}(x)|$ p=+∞ bo'ladi.

$f(x) = \frac{a}{x^2+1}$ funksiyaning p-adik dinamik sistemalari.

p-Adik son.

Q- ratsional sonlar maydoni bo'lsin. n va m sonlarning EKUB (n,m) bo'lsin. Har bir x ≠0 ratsional sonni $x = p^r \frac{n}{m}$ ko'rinishda yozish mumkin. Bu yerda r,n∈Z, m∈N, (p,n)=1,(p,m)=1 va p fiksirlangan tub son.

x ning p-adik normasi quyidagicha aniqlanadi:

$$|x|_p = \begin{cases} p^{-r}, & \text{agar } x \neq 0 \\ 0, & \text{agar } x = 0 \end{cases}$$

p-Adik norma xossalari:

1) $|x|_p \geq 0$ va $|x|_p = 0$ faqat va faqat x=0 bo'lsa.

2) $|xy|_p = |x|_p|y|_p$.

3)Qat'iy uchburchak tengsizligi

$|x+y|_p \leq \max\{|x|_p, |y|_p\}$

3.1) Agar $|x|_p \neq |y|_p$ bo'lsa, u holda $|x+y|_p = \max\{|x|_p, |y|_p\}$

3.2) Agar $|x|_p = |y|_p$ bo'lsa, u holda p=2 uchun $|x+y|_p \leq 0{,}5|x|_p$ o'rinli.

Ma`lumki p-adik norma $|.|_p$ ga nisbatan Q ratsional sonlar to`plamining to'ldirmasi (barcha limit nuqtalari to'plami) Q_p orqali belgilanadigan p-adik maydonni aniqlaydi.

Q_p ning algebraik to'ldirmasi kompleks p-adik sonlar maydoni deb ataladi va u C_p kabi belgilanadi.

Ixtiyoriy $a \in C_p$ va r>0 uchun quyidagi to'plamlarni qaraylik

$U_r(a) = \{x \in C_p : |x-a|_p < r\}$,

$V_r(a) = \{x \in C_p : |x-a|_p \leq r\}$,

$S_r(a) = \{x \in C_p : |x-a|_p = r\}$.

$f: U_r(a) \to C_p$ funksiyani $U_r(a)$ sharda quyidagi tekis yaqinlashuvchi $f(x) = \sum_{n=0}^{\infty} f_n(x-a)^n$, $f_n \in C_p$, qator ko'rinishida yozish mumkin bo'lsa, unga analitik funksiya deyiladi.

C_p da dinamik sistemalar.

C_p dagi (f, U) dinamik sistemasining ba'zi xossalarini keltirib o'tamiz. Bu yerda $f: x \in U \to f(x) \in U$ analitik funksiya va $U = U_r(a)$ yoki C_p.

$f: U \to U$ analitik funksiya bo'lsin. $f^n(x) = f * \ldots * f(x)$ kabi belgilash kiritamiz.

Agar $f(x_0) = x_0$ bo'lsa, x_o –qo'zg'almas nuqta deyiladi. f ning barcha qo'zg'almas nuqtalari to'plami *Fix*(f) kabi belgilanadi. Agar x_o ning shunday $U(x_o)$ atrofi topilib, barcha $x \in U(x_0)$ nuqtalar uchun

$\lim_{n\to\infty} f^n(x) = x_0$ bo'lsa, x_0 tortuvchi deyiladi. Agar x_0 tortuvchi bo'lsa, tortishishlar to'plami quyidagicha bo'ladi:

A(x_0)=$\{x \in C_p : f^n(x) \to x_0, n \to \infty\}$.

Agar x_0 nuqtaning shunday U(x_0) atrofi mavjud bo'lib, barcha x∈ $U(x_0), x \neq x_0$ nuqtalarda $|f(x) - x_0|_p > |x - x_0|_p$ tengsizlik o'rinli bo'lsa, x_0 itaruvchi deyiladi.

$\lambda = f`(x_0)$ bo`lsin. Agar $0 < |\lambda|_p < 1$, bo`lsa $x_0$ ga tortuvchi nuqta, $|\lambda|_p = 1$ bo`lsa netral nuqta va $|\lambda|_p > 1$ bo`lsa itaruvchi nuqta deyiladi.

$\underline{U}_r$ (x_0) shar Siegel diski deyiladi, agar uning har bir sferasi $S_\rho(x_0)$, $\rho < r$ f(x) ning invariant sferasi bo`lsa, yani agar x ∈ $S_\rho(x_0)$ bo`lsa, u holda f^n(x) ∈ $S_\rho(x_0)$ bo`ladi bu yerda n = 1, 2 . . . . Barcha Siegel disklarining ($x_0$ markazli) birlashmasi maksimal Siegel diski deyiladi va SI ($x_0$) kabi belgilanadi . $f$ funksiyaga bog`liq dinamik sistemani qaraymiz: f : $C_p \to C_p$ bu Sistema quyidagicha aniqlangan.

f:U→U va g:V→V akslantirishlar berilgan bo'lsin. h:U→V gomeomorfizm mavjud bo'lib, h°f=g°h bo'lsa, f va g akslantirishlar topologik qo'shma akslantirishlar deyiladi. h–gomeomorfizmga topologik qo'shmalik deyiladi. Topologik qo'shma akslantirishlar o'zining dinamikasi bo'yicha to'la ekvivalent bo'ladi. Misol uchun, agar f akslantirish g akslantirish bilan h orqali qo'shma, x_0 esa f uchun qo'zg'almas nuqta bo'lsa, u holda h(x_0) g akslantirish uchun qo'zg'almas bo'ladi. Haqiqatdan ham, h(x_0) =h f(x_0)=gh(x_0).

$a/(x^2+1)$ funksiya.

Endi biz f(x)= a/(x^2+1), $a \neq 0$, $a \in C_p$ $x^2 \neq -1$ (2.1)

tenglik bilan aniqlangan f : $C_p \to C_p$ funksiya bilan bog'langan dinamik sistemani qaraymiz.

3.3.1 Eslatma. x^2=-1 tenglama $x \in Q_p$ yechimiga ega bo'lishi uchun p=1(mod4) bo'lishi zarur va yetarli.

Biz qarayotgan (2.1) tenglik bilan aniqlangan funksiya C_p yopiq to'plamda algebraik bo'lganligi uchun, bizning holda x^2=-1 (p tup songa bog'liqsiz holda) ikkita $\pm i$ yechimga ega.

Bizning asosiy maqsadimiz komleks p-adik maydon C_p da (2.1) tenglik bilan aniqlangan funksiya trayektoriyasini o'rganish.

Bu funksiya ($a \neq 0$) uchun 3 ta qo'zg'almas nuqtaga ega:

f(x)=x x^3+x-a=0 x_k=x_k(a), k=1,2,3. (2.2)

Har bir yechimning aniq ko'rinishini berish mumkin, ammo yechimlarning ko'rinishi qo'pol ko'rinishda ega bo'lishi mumkin. Umumiylikni chegaralamagan holda

$|x_1|_p \leq |x_2|_p \leq |x_3|_p$ (2.3)

bo'lsin deb faraz qilamiz.

A=$|a|_p$ kabi belgilash kiritamiz.

Viyet formulasiga ko'ra:

$x_1+x_2+x_3=0$, $x_1x_2x_3=a$ (2.4)

3.3.1 Lemma. x_k k=1,2,3 qo'zg'almas nuqtalarning normasi uchun quyidagilar o'rinli:

$$|x_1|_p = A, \quad |x_2|_p = |x_3|_p = 1, \quad agar \quad A \leq 1$$

$$|x_1|_p = |x_2|_p = |x_3|_p = A^{\frac{1}{3}}, \quad agar \quad A > 1.$$

Isbot. x_k nuqta f ning qo'zg'almas nuqtasi bo'lganligi uchun

$$|x_k|_p = \left|\frac{a}{x_k^2+1}\right|_p = \begin{cases} \text{A}, & \text{agar } |x_k|_p < 1 \\ \geq \text{A}, & \text{agar } |x_k|_p = 1 \\ \dfrac{A}{|x_k|_p^2}, & agar\ |x_k|_p > 1 \end{cases}$$

Bu sistemani yechib bo'ladi.

$$|x_k|_p = \begin{cases} \text{A, agar A} \leq 1 \\ A^{1/3}, \text{ agar } A > 1 \end{cases} \qquad (2.5)$$

ega bo'lamiz.

A ≤ 1 bo'lgan hol. $|x_1|_p = A$ bo'lsa, (2.4) tenglikdan $|x_2x_3|_p = 1$ va (2.5) dan $|x_2|_p = |x_3|_p = 1$ munosobatlarni hosil qilamiz. Bundan tashqari, (2.3)-(2.5) ga ko'ra $|x_1|_p = 1$ bo'lishi mumkin emas.

$A > 1$ bo'lgan hol. Bu holda (2.5) dan $|x_1|_p = |x_2|_p = |x_3|_p = A^{\frac{1}{3}}$ ni hosil qilamiz.

3.3.2 Lemma. Quyidagi munosabatlar o'rinli:

1.Agar p=2, A≤ 1 bo'lsa, u holda x_1 tortuvchi,

x_2 va x_3 $\begin{cases} \text{tortuvchi, agar } 0,5 < A \leq 1 \\ \text{neytral, agar } A = 0,5 \\ itaruvchi, agar\ A < 0,5 \end{cases}$

2.Agar p$\geq 3, A < 1$ bo'lsa, u holda x_1 tortuvchi x_2 va x_3 itaruvchi bo'ladi.

3.Agar p$\geq 3, A = 1$ bo'lsa, u holda x_i, i=1,2,3 neytral bo'ladi.

4. Agar A> 1 bo'lsa, u holda

x_i $\begin{cases} \text{tortuvchi, agar p} = 2 \\ \text{neytral, agar } p \geq 3 \end{cases}$

Isbot.

$$f'(x) = -2x \cdot \frac{a}{(x^2+1)^2} = -\frac{2x}{a} \cdot \left(\frac{a}{x^2+1}\right)^2 = -\frac{2x}{a}(f(x))^2$$

tenglik o'rinli.

Bundan

$$f'(x_k) = -\frac{2x_k}{a}\left(f(x_k)\right)^2 = -\frac{2x_k^3}{a}$$

tenglik hosil bo'ladi. Ushbu tenglik va lemma 3.3.1 dan lemma 3.3.2 ning isboti kelib chiqadi.

x$\in S_r(0)$, ya'ni r= $|x|_p$ uchun (3) formula va p-adik normaning qat'iy uchburchak tengsizligidan quyidagi munosabatni hosil qilamiz:

$$r' = |f(x)|_p = \varphi(x) = \begin{cases} \text{A, agar } r < 1 \\ \geq \text{A, agar r} = 1 \\ \frac{A}{r^2}, agar\ r > 1 \end{cases} \tag{2.6}$$

3.3.2 Eslatma. r=1 da φ(r) aniq ko'rinishga ega emas. Bizga faqatgina uning quyi chegarasi aniq. Quyida biz keltiradigan mulohazalarda (r=1 bo'lganda) bu quyi chegara topiladi.

$$P = \left\{x \in C_p: \exists n \in N \cup \{0\}, f^n(x) \in \{-i, i\}\right\}, \tag{2.7}$$

belgilash kiritamiz.

3.3.3 Lemma. Agar x ∈ $S_r(0)$ va x∉P bo'lsa, u holda har bir n≥ 1uchun

$$\left|f^{(n)}(x)\right|_p = \varphi^n(r)$$

tenglik o'rinli.

Isbot. n=1 bo'lsa (2.6) formuladan tenglik isboti kelib chiqadi. Endi biz n=2 bo'lgan holni qaraymiz. $|f(x)|_p = \varphi(r)$ bo'lganligi uchun:

$$|f^2(x)|_p = \frac{|a|_p}{|(f(x))^2+1|_p} = \varphi(\varphi(r)) = \begin{cases} \text{A, agar } \varphi(r) < 1 \\ \geq \text{A, agar } \varphi(r) = 1 \\ \frac{A}{(\varphi(r))^2}, agar\ \varphi(r) > 1 \end{cases}$$

tenglikga ega bo'lamiz.

Ushbu mulohazalarni ixtiyoriy n≥1 va barcha $x \in S_r(0) \setminus P$ uchun qo'llab lemmani to'la isbotlaymiz.

3.3.4 Lemma. $\varphi(r)$ ((2.6) tenglik bilan aniqlangan) hosil qilgan dinamik sistema quyidagi xossalarga ega:

1. $Fix(\varphi) = \begin{cases} \{\text{A}\} \cup \{\text{1: agar } \varphi(1) = 1\}, \text{barcha A} < 1 \\ \{\text{1: agar } \varphi(1) = 1\}, \text{ barcha A} = 1 \\ \{\sqrt[3]{A}\}, barcha\ A > 1. \end{cases}$

2. Agar $A \leq 1$ bo'lsa, u holda

- barcha r< 1 uchun

$$\varphi(r) = A, \qquad \varphi(A) = A$$

- agar $A \leq \varphi(1)$< 1 bo'lsa, u holda $\varphi^n(1) = A,\ n \geq 2.$

- agar A< $\varphi(1)$bo'lsa, u holda $\varphi^2\ (1) = \frac{A}{(\varphi(1))^2}, \varphi^n(1) = A,\ n \geq 3.$

r> 1 $uchun$

$\varphi(r) = \frac{A}{r^2}$, φ^n (r)=A, $n \geq 2.$

3. Agar A> 1 bo'lsa u holda {A,1/A} φ uchun r davriy orbita va

-har bir r< 1 uchun

$$\varphi(r) = A, \qquad \varphi(A) = A.$$

-agar r=1 bo'lsa, u holda $\varphi^2(1) = \frac{A}{(\varphi(1))^2}, \varphi^n(1) = A,\ n \geq 3.$

-agar r> 1 $r \neq \sqrt[3]{A}$ bo'lsa, u holda

$$\lim_{k\to\infty} \varphi^{2k}(r) = \frac{1}{A}, \qquad \varphi^{2k+1}(r) = A, \quad k = 0,1,\dots$$

Isbot. (2.6) tenglik bilan aniqlangan $\varphi: [0;+\infty) \to [0;+\infty)$ funksiyaning sodda xossalaridan lemmaning isboti kelib chiqadi.

3.3.1 Teorema. (2.1) tenglik yordamida aniqlangan funksiya hosil qilgan dinamik sistema quyidagi xossalarga ega:

1. Agar A≤1 bo'lsa, u holda $f(S_A(0)) \subset S_A(0)$ va har bir x ∈ $S_r(0)$ uchun

-barcha r <1 larda,

$f^n(x) \in S_A(0)$, n≥ 1.

Agar r=1 va $|f(x)|_p \leq 1$ bo'lsa, u holda $f^n(x) \in S_A(0)$, n≥ 2.

-Agar $|f(x)|_p > 1$ bo'lsa, u holda $f^2(x) = S_{\frac{A}{\{|f(x)|_p\}^2}}$, $f^n(x) = S_A(0)$ n ≥ 3.

-Agar r> 1 bo'lsa, u holda

$f(x) \in S_{\frac{A}{r^2}}(0)$. $f^n(x) \in S_A(0)$. n ≥ 2.

2.Agar A > 1 bo'lsa, u holda $f(S_{1/A}(0)) \subset S_A(0)$. $f(S_A(0)) \subset S_{1/A}(0)$ va barcha r< 1 larda

$$f^n(x) \subset S_A(0),\ n \geq 1.$$

-agar r=1 bo'lsa, u holda $f^2(x) \in S_{\frac{A}{(f(x)p)^2}}$, $f^n(x) \in S_A(0)$ n ≥ 3.

-agar r>1 bo'lsa, u holda $f(S_{\sqrt[3]{A}}(0)) \subset (S_{\sqrt[3]{A}}(0))$ va bunda r≠ $\sqrt[3]{A}$ bu holda

$$\lim_{k\to\infty} |f^{2k}(x)|_p = \frac{1}{A}, \qquad f^{2k+1}(x) \in S(0), \quad k = 0,1,\dots$$

Bu teorema $f^n(x)$ ketma-ketlikning limitini bermaydi. Bu limitni o'rganish uchun quyidagi teoremadan foydalanamiz.

3.3.2 Teorema. x_o nuqta $f: U \to U$ analitik funksiya uchun qo'zg'almas nuqta bo'lsin. Quyidagi tasdiqlar o'rinli bo'ladi:

1. Agar x_o nuqta f uchun tortuvchi nuqta $r > 0$

$$Q = \max_{1\leq n<\infty} \left|\frac{1}{n!}\frac{d^n f}{dx^n}(x_0)\right|_p r^{n-1} < 1 \qquad (2.8)$$

tenglikni qanoatlantirsa va $U_r(x_0) \subset U$ bo'lsa, u holda $U_r(x_0) \subset A(x_0)$ bo'ladi;

2.Agar x_0 nuqta f uchun neytral nuqta bo'lsa, u holda bu nuqta siegel diskining markazi bo'ladi. Agar r

$$S = \max_{2\leq n<\infty} \left|\frac{1}{n!}\frac{d^n f}{dx^n}(x_0)\right|_p r^{n-1} < |f'(x_0)|_p \qquad (2.9)$$

tenglikni qanoatlantirsa va $U_r(x_0) \subset U$ bo'lsa, u holda $U_r(x_0) \subset SI(x_0)$ bo'ladi.

(2.1) tenglik bilan aniqlangan funksiya uchun (2.8) ning yechimini r_o orqali, (2.9) ning yechimini r_1 orqali belgilaymiz. U holda 3.3.2- teorema, 3.31,3.3.2-lemmalar va 3.3.1- teoremadan quyidagi natijani olamiz:

3.3.3 Teorema. Har bir tortuvchi qo'zg'almas yoki davriy nuqtalar to'plami r_o radiusli tortishishlar to'plamida yotuvchi ochiq shardan iborat.

Barcha neytral qo'zg'almas nuqtalar to'plami r_1 radiusli Siegel diskining markazi bo'ladi.

XULOSA

$f(x) = \frac{a}{x^2}$ funktsiyasining p-adik kompleks sonlar maydoni ustida dinamik sistemasi o'rganildi.

Bu dinamik sistema uchun qo'zg'almas va davriy nuqtalar mavjudlik shartlari va ularning turlari aniqlandi. Parameterlarga qo'yilgan ba'zi shartlar asosida traektoriyalarning limit nuqtalar to'plami va funksiyaning Siegel disklari topildi.

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